Problem suggestions (group 1, 9.45-11.15)

Rules. At least one of the examples below will appear on the long test. You can try to do it yourself (this is a good idea), ask your colleagues who proposed these problems (also a good idea) and use whatever tools you have, e.g. Wolfram Alpha.

However, remember that no auxiliary tools (notes, smartphones etc.) will be available during the test.

What exactly is the problem? You can assume the problem has one of these two forms.

1. Given the matrix A, find its eigenvalues and the corresponding eigenspaces. Decide whether A is diagonalizable. If it is, find a diagonal matrix D and an invertible matrix C such that $A = CDC^{-1}$.

2. Given the linear map $\varphi\colon \mathbb{R}^n \to \mathbb{R}^n$ defined by $[\varphi]_{\mathrm{st}}^{\mathrm{st}} = A$, find its eigenvalues and the corresponding eigenspaces. Decide whether φ is diagonalizable. If it is, find a basis $\mathcal{A}$ such that $[\varphi]_{\mathcal{A}}^{\mathcal{A}}$ is diagonal, and compute this diagonal matrix.

Problem 1.

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 5 & 5 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Problem 2.

$$A = \begin{bmatrix} 3 & 3 & -3 \\ -2 & 4 & 2 \\ -1 & 5 & 1 \end{bmatrix}$$

Problem 3.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -2 & 4 & 2 \\ -1 & 1 & 5 \end{bmatrix}$$

Problem 4.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

Problem 5.

$$A = \begin{bmatrix} 1 & 3 & 18 \\ 3 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 6.

$$A = \begin{bmatrix} 4 & 5 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$